

# Formalizing and Implementing Types in MSVL

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**Abstract.** This paper investigates techniques for formalizing and implementing types in the temporal logic programming language MSVL, which is an executable subset of Projection Temporal Logic. To this end, the data domain of MSVL is extended to include typed values, and then typed functions and predicates concerning the extended data domain are defined. Based on these definitions, the statement for type declaration of program variables is formalized. The implementation mechanisms of the type declaration statement in the MSVL interpreter are also discussed, which is based on the notion of normal form of MSVL programs. To illustrate how to program with types, an example of in-place reversing an integer list is given.

**Keywords:** Type · Temporal logic programming · MSVL · Projection Temporal Logic

## 1 Introduction

The paper is organized as follows. Section 2 introduces the basic concepts of MSVL. Section 3 formalizes the type declaration statement. Section 4 discusses the implementation mechanisms of the type declaration statement. Section 5 illustrates how to program with types by an example of in-place reversing an integer list. Section 6 concludes the paper.

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$(e)$  temporal  $(e)$   
 $\mathbf{A} s, e s s, s s, s (I_v, I_p) w e e, e e, e x e, e s$   
 $s x I_v x, e r e, s s r I_p r I_v x s, e$   
 $e D nil w e s, e e, I_p r s, e$   
 $\mathbb{B} \{true, false\} \mathbf{A} e, \sigma \langle s_0, s_1, \dots \rangle s, e (ss, e)$   
 $s e e, s, e e, \sigma e e, |\sigma| s e e, s w, \sigma s, e$   
 $e w s e, e, e, s, \sigma s, e, e,$   
 $s w e, s e, e s e N, e s, \omega N_w N \cup \{\omega\},$   
 $e e, e, s, e, s, <, \leq N_w, s e, \omega w,$   
 $i \in \mathbb{N} i < \omega, e e, w e, e \leq, s \leq -\{(\omega, \omega)\} s,$   
 $\sigma(i..j) (\leq i \preceq j \leq |\sigma|) e, e s, e, \langle s_i, \dots, s_j \rangle, \sigma(k) (\leq k \preceq |\sigma|)$   
 $e, e s \langle s_k, \dots, s_{|\sigma|} \rangle e, e, \sigma w, e, e, (e,$   
 $s) \sigma' s e, e, \sigma \bullet \sigma', e e e s e, e, e,$   
 $w e, e e, e, e, s e \sigma \langle s_0, s_1, \dots \rangle e, e,$   
 $n_1, \dots, n_h e, e e s (h \geq) s, \leq n_1 \leq n_2 \leq \dots \leq n_h \preceq |\sigma|$   
 $e, e, \sigma, n_1, \dots, n_h s, e, e, (e e e e, e) \sigma \downarrow$   
 $(n_1, \dots, n_h) \langle s_{m_1}, s_{m_2}, \dots, s_{m_l} \rangle w e e m_1, \dots, m_l s, e, \dots, n_1, \dots, n_h$   
 $e e, e s, e, e$

$$\langle s_0, s_1, s_2, s_3, s_4 \rangle \downarrow (, , , , ) \langle s_0, s_2, s_3 \rangle$$

$\mathbf{A} e, e, e, s, e \mathcal{I} (\sigma, i, k, j) w e e$   
 $\sigma \langle s_0, s_1, \dots \rangle s, e, i, k, e, e e s, j s,$   
 $e e, \omega s, i \leq k \preceq j \leq |\sigma| e s e (\sigma, i, k, j) e, e,$   
 $s, e e e, s, e, \sigma(i..j) w e e s, e e, s_k$   
 $e e, e e e, e e, e e, e e, e e, \mathcal{I} (\sigma, i, k, j)$   
 $e e, s \mathcal{I} e s, e D nil s e e, s, e$   
 $s w,$

$$\begin{aligned}
 \mathcal{I}[x] &= s_k[x] = I_v^k[x] \\
 \mathcal{I}[\bigcirc e] &= \begin{cases} (\sigma, i, k+1, j)[e] & \text{if } k < j \\ nil & \text{otherwise} \end{cases} \\
 \mathcal{I}[\ominus e] &= \begin{cases} (\sigma, i, k-1, j)[e] & \text{if } i < k \\ nil & \text{otherwise} \end{cases} \\
 \mathcal{I}[f(e_1, \dots, e_m)] &= \begin{cases} \mathcal{I}[f](\mathcal{I}[e_1], \dots, \mathcal{I}[e_m]) & \text{if } \mathcal{I}[e_h] \neq nil \text{ for all } h \\ nil & \text{otherwise} \end{cases}
 \end{aligned}$$

Fig. 1. Interpretation of PTL terms

$$\begin{aligned}
 \mathcal{I} \mid r, s_k r, I_p^k r, true \\
 \mathcal{I} \mid e_1, e_2, \mathcal{I} e_1, \mathcal{I} e_2
 \end{aligned}$$





$\square$	$\neg$	$\circ$	$\ominus$	$\square$	3	$*$	$/$	$mod$	4	$+$	$-$	5	$>$	$\geq$	$<$	$\leq$	6	$\exists$	
$\blacktriangle$	$\Leftarrow$	$:=$	8	$\wedge$	9	$\vee$	$\parallel$	10	$\rightarrow$	$\leftrightarrow$	11	$prj$	12						

Table 2. Precedence rules of MSVL

### 3 Typed MSVL

#### 3.1 Data Domain

$\mathcal{D} = \langle I_v, I_p \rangle$ ,  $I_v x \in \mathcal{D}$ ,  $I_p y \in \mathcal{D}$ ,  $(I_v x, I_p y) \in \mathcal{D}$ ,  $nil \in \mathcal{D}$ ,  $\mathcal{I} = \{ I_v, I_p, nil \}$ ,  $\mathcal{D} = \{ I_v, I_p, nil \}$ ,  $\mathcal{T} = \{ \langle \rangle, \langle \rangle \}$ ,  $\mathbb{Z}$ ,  $\mathbb{F} \stackrel{def}{=} \{ n.d_1d_2 \dots d_m \mid m \in \mathbb{N}, n \in \mathbb{Z}, d_i \in \{ \dots \} \text{ for } 1 \leq i \leq m \}$ ,  $\mathbb{C} \stackrel{def}{=} \{ 'a', \dots, 'z', 'A', \dots, 'Z', ' ', \dots, '!', '\$', \dots \}$ ,  $\mathcal{S} = \langle v, T \rangle$ ,  $\mathcal{S}^n = \langle S, S^n \rangle$ ,  $\mathcal{S}^* = \langle S^*, S^* \rangle$



...  $e$  ...  $e$  **A** ...  $e$   $\check{e}$  ...  $e$   $e$  ...  $\check{e}$   $s$  ...  $e$   $w$  ...  
 $e$  ...  $s$  ...  $e$

$$\cdot \perp, (\cdot \times \cdot \rightarrow \cdot) \cup (\cdot \times \cdot \rightarrow \cdot)$$

...  $s$   $w$  ...  $w$  ...  $\check{e}$  ...  $f$  ( ...  $e$   $\check{e}$   $e$   $P$ ) ...  $e$  ...  $e$  ...  
 $e$   $e$  ...  $e$   $s$  ...  $\check{e}$  ...  $\check{e}$  ...  $f$  ( ...  $P$ ) ...  $e$   $s$  ...  $e$  ...  $e$   $s$  ...  $e$   $s$

*Arithmetic operators.*  $e$   $s$   $e$   $s$   $\perp$   $w$  ...  $e$   $s$  ...  $e$   $e$   $e$   $s$  ...  $-$   $*$  ...  $/$   $s$  ...

$$\cdot \perp, \cdot - \cdot, \cdot * \cdot, \cdot / \cdot, (\cdot \times \cdot \rightarrow \cdot) \cup (\cdot \times \cdot \rightarrow \cdot)$$

$e$   $e$  ...  $e$  ...  $\check{e}$  ...  $mod$ , ...  $\times$  ...  $\rightarrow$  ...  $e$ ,  $s$  ...  $e$   $e$  ...  $e$   $e$   $\check{e}$   $e$   $s$   
 $>$ ,  $\geq$ ,  $<$ ,  $\leq$  ...  $e$   $e$  ...  $e$   $s$

$$\cdot > \cdot, \cdot \geq \cdot, \cdot < \cdot, \cdot \leq \cdot, (\cdot \times \cdot \rightarrow \mathbb{B}) \cup (\cdot \times \cdot \rightarrow \mathbb{B})$$

*Type cast.*  $e$   $e$   $e$   $w$  ...  $\check{e}$  ...  $s$  ...  $e$   $\check{e}$   $s$  ...  $w$   $e$  ...  $e$   $s$  ...  
 $e$   $e$   $s$

$$\begin{aligned} (\cdot) \cdot, \cdot \rightarrow \cdot & \quad (n.d_1d_2 \cdots d_m, \cdot) \mapsto (n, \cdot) \\ (\cdot) \cdot, \cdot \rightarrow \cdot & \quad (n, \cdot) \mapsto (n, \cdot) \end{aligned}$$

*Array and list operations.*  $e$   $e$   $e$   $s$   $e$  ...  $s$  ...  $e$  ...  $s$  ...  $s$  ...  
 $s$   $s$  ...  $a$  ...  $e$  ...  $a$   $i$  ...  $s$   $s$   $i$  ...  $e$   $e$

$$\begin{aligned} \cdot \cdot, (\cdot \times \cdot \rightarrow \cdot) \cup (\cdot \times \cdot \rightarrow \cdot) \cup (\check{\cdot} \times \cdot \rightarrow \check{\cdot}) \\ \langle (c_0, \dots, c_k), T \rangle, (i, \cdot) \mapsto (c_i, T) \quad i \in \{ \cdot, \dots, k \} \\ \langle (c_0, \dots, c_k), T \rangle, (i, \cdot) \mapsto nil \quad i \notin \{ \cdot, \dots, k \} \\ k \in \mathbb{N}, T \in \{ \cdot, \dots, \check{\cdot} \} \end{aligned}$$

...  $s$   $l$  ...  $e$  ...  $|l|$  ...  $s$  ...  $e$   $e$  ...  $l$

$$\begin{aligned} |\cdot|, (\cdot \langle \rangle \rightarrow \cdot) \cup (\cdot \langle \rangle \rightarrow \cdot) \cup (\check{\cdot} \langle \rangle \rightarrow \cdot) \\ \langle (c_1, \dots, c_k), T \rangle \mapsto (k, \cdot) \quad k \in \mathbb{N}, T \in \{ \cdot \langle \rangle, \dots \langle \rangle, \check{\cdot} \langle \rangle \} \end{aligned}$$

$e$   $s$   $e$   $s$  ...  $e$  ...  $s$   $hd(l)$ , ...  $tl(l)$  ...  $e$  ...  $e$  ...  $l$  ...  $e$   $\check{e}$  ...  $e$

$$\begin{aligned} hd, (\cdot \langle \rangle \rightarrow \cdot) \cup (\cdot \langle \rangle \rightarrow \cdot) \cup (\check{\cdot} \langle \rangle \rightarrow \check{\cdot}) \\ \langle \rangle, T \langle \rangle \mapsto nil \end{aligned}$$

$$\langle (c_0, c_1, \dots, c_k), T \langle \rangle \rangle \mapsto (c_0, T) \quad k \in \mathbb{N}, T \in \{ \cdot, \dots, \check{\cdot} \}$$

$$\begin{aligned} tl, (\cdot \langle \rangle \rightarrow \cdot \langle \rangle) \cup (\cdot \langle \rangle \rightarrow \cdot \langle \rangle) \cup (\check{\cdot} \langle \rangle \rightarrow \check{\cdot} \langle \rangle) \\ \langle \rangle, T \mapsto nil \end{aligned}$$

$$\langle (c_0, c_1, \dots, c_k), T \rangle \mapsto \langle (c_1, \dots, c_k), T \rangle \quad k \in \mathbb{N}, T \in \{ \cdot \langle \rangle, \dots \langle \rangle, \check{\cdot} \langle \rangle \}$$

...  $w$  ...  $s$   $s$   $l_1$ , ...  $l_2$  ...  $e$   $s$  ...  $e$  ...  $e$  ...  $s$   $l_1 \bullet l_2$ , ...  $l_1 \circ l_2$   $\check{e}$  ...  
 $\check{e}$  ...  $e$   $s$  ...  $e$   $\check{e}$  ...  $e$  ...  $s$  ...  $l_1$ , ...  $l_2$  ...  $e$   $\check{e}$  ...  $e$  ...  $e$   $e$   $e$   $e$  ...  
 $s$  ...  $w$   $s$

$$\begin{aligned} \cdot \bullet \cdot, (\cdot \langle \rangle \times \cdot \langle \rangle \rightarrow \cdot \langle \rangle) \cup (\cdot \langle \rangle \times \cdot \langle \rangle \rightarrow \cdot \langle \rangle) \\ \cup (\check{\cdot} \langle \rangle \times \check{\cdot} \langle \rangle \rightarrow \check{\cdot} \langle \rangle) \\ \langle (c_1, \dots, c_j), T \rangle, \langle (d_1, \dots, d_k), T \rangle \mapsto \langle (c_1, \dots, c_j, d_1, \dots, d_k), T \rangle \\ j, k \in \mathbb{N}, T \in \{ \cdot \langle \rangle, \dots \langle \rangle, \check{\cdot} \langle \rangle \} \end{aligned}$$





*(The text in this block is extremely faint and largely illegible, appearing to be a series of characters and symbols.)*

### 4.1 Normal Form of Programs

**Definition 1.** A typed MSVL program  $q$  is in normal form if

$$q \stackrel{\text{def}}{=} \left( \bigvee_{i=1}^k q_{ei} \wedge \text{empty} \right) \vee \left( \bigvee_{j=1}^h q_{cj} \wedge \bigcirc q_{fj} \right)$$

where  $k + h \geq 1$  and the following hold:

1. each  $q_{ei}$  and  $q_{cj}$  is either true or a state formula of the form  $p_1 \wedge \dots \wedge p_m$  ( $m \geq 1$ ) such that each  $p_l$  ( $1 \leq l \leq m$ ) is either  $s_T(x)$  with  $x \in \mathcal{V}$ ,  $T \in \mathcal{T}_d$ , or  $x = e$  with  $e \in D$ , or  $r_x$ , or  $\neg r_x$ .
2.  $q_{fj}$  is an internal program, that is, one in which variables may refer to the previous states but not beyond the first state of the current interval over which the program is executed.

*(The text in this block is extremely faint and largely illegible, appearing to be a series of characters and symbols.)*

$$\begin{aligned}
 Tx &\equiv \square s_T(x) \\
 &\equiv \square s_T(x) \wedge (\text{empty} \vee \neg \text{empty}) && (i) \\
 &\equiv \square s_T(x) \wedge \text{empty} \vee \square s_T(x) \wedge \neg \text{empty} && (ii) \\
 &\equiv s_T(x) \wedge \text{empty} \vee \square s_T(x) \wedge \neg \text{empty} && (iii) \\
 &\equiv s_T(x) \wedge \text{empty} \vee \square s_T(x) \wedge \text{more} && (iv) \\
 &\equiv s_T(x) \wedge \text{empty} \vee s_T(x) \wedge \bigcirc \square s_T(x) && (v)
 \end{aligned}$$

... e, e ( ) ... ws ... ( ) ... e.e ... ( )  
 ... w ... ( ) ... e.e ... more ... ( ) ... w

### 4.2 MSVL Interpreter

... S, S, L, S, ee, Se ... e.e ... e.e.e ...  
 ... w ... e.e.e.e.e.S.S.w ... e.e.e ... Se.  
 ... e.e.e.e.w ... S ... e.S ... Se ...  
 ... e.e.e.e.w.e ... e.e.e ...  
 ... Present  $\wedge$  Remains ... Present ...  
 ... S, e ... SSS, true false empty ... SS, e.S ...  
 ... e.e.e.e.e ... Remains ...  
 ... S, e.e.e.e.e.S.S ... e.w ...  
 ... S, e.Se ... e.w ...  
 ... e.e.e.e.e.e.e ... S, e.S ...  
 ... e ... S ... S ... true ... S, e.S ...  
 ... S.S, S, e.w.Se ... S ... S.S, S, e

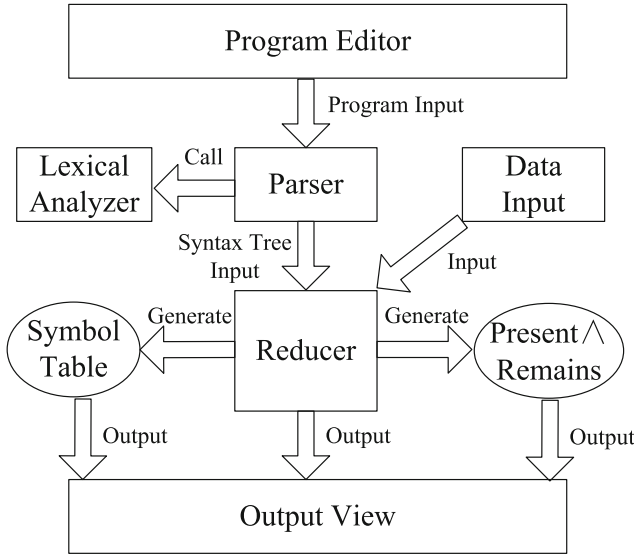
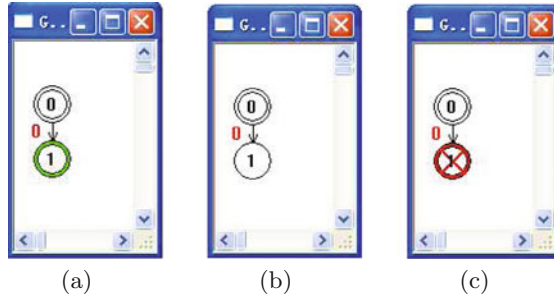
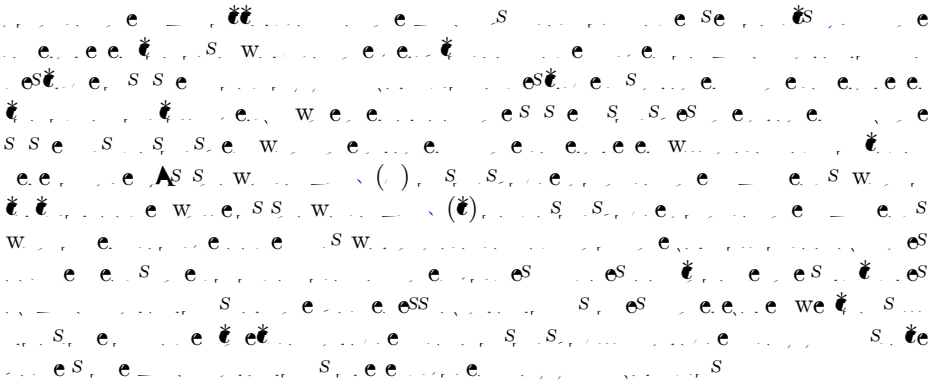


Fig. 3. Interpreter structure

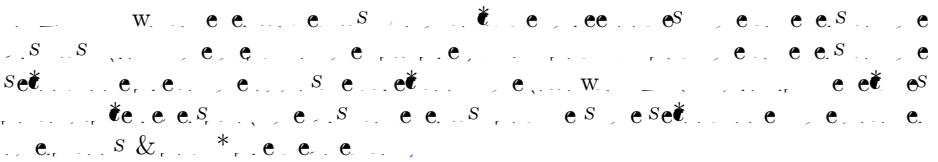
... e ... e.e.e.S, e.w ... e.S ...  
 ... e.e.e.S ... S.Se ... e.S.S.S ...  
 ... e.e.e.e.e.e.e. ... e.S ... e.S.S.e ...  
 ... ( ) ... S.S.w ... ( ) ... e ...  
 ... S.w ... e.S, ... e.S.S.e ... e.S, ...  
 ... e.e.e.w ... e.e.e.e.e.e.e.S ... e



**Fig. 4.** Three types of nodes. (a) modeling: a path. (b) verification: a satisfiable path. (c) verification: an unsatisfiable path.



### 5 An Application



```

frame(node1, node2, node3, p, q, r, head, tail) and (
    int[2] node1, node2, node3;
    pointer p,q,r;
    node1[0] = 10 and node1[1] := -1;
    node2[0] = node1[0] + 10 and node2[1] := -1;
    node3[0] = (int)30.0 and node3[1] := -1;
    node2[1] := &node3;    node1[1] := &node2;
    q :=& node1;    head := *q[0];
    while(q != -1){ tail := *q[0];    q := *q[1] };
    p :=& node1;    q := -1;

```



